C3 Differentiation

1. June 2010 qu. 1

Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3e^{2x}$, [2]

(ii) $y = \ln(3 + 2x^2)$, [2]

(iii) $y = \frac{x}{2x+1}$. [2]

2. Jan 2010 qu.5

The equation of a curve is $y = (x^2 + 1)^8$.

(i) Find an expression for $\frac{dy}{dx}$ and hence show that the only stationary point on the curve is the point for which $x = 0$. [4]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence find the value of $\frac{d^2y}{dx^2}$ at the stationary point. [5]

3. Jan 2010 qu. 7

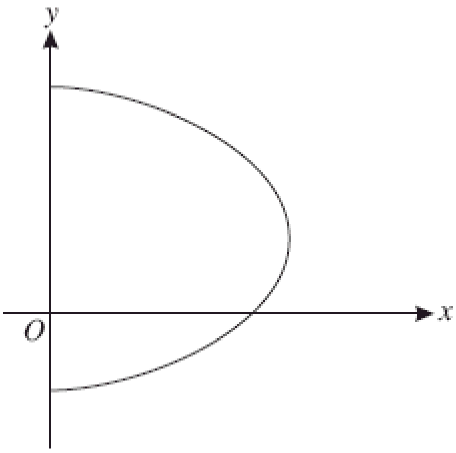
(a) Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant when the area of the patch is 1900 square metres. Give your answer correct to 2 significant figures. [4]

(b) The mass of a substance is decreasing exponentially. Its mass now is 150 grams and its mass, m grams, at a time t years from now is given by

$$m = 150e^{-kt},$$

where k is a positive constant. Find, in terms of k, the number of years from now at which the mass will be decreasing at a rate of 3 grams per year. [3]

4. June 2009 qu. 6



The diagram shows the curve with equation $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. [2]

(ii) Hence find the equation of the tangent to the curve at the point (7, 3), giving your answer in the form $y = mx + c$. [5]

5. June 2009 qu. 9

(a) Show that, for all non-zero values of the constant k, the curve $y = \dfrac{kx^2 - 1}{kx^2 + 1}$ has exactly one stationary point. [5]

(b) Show that, for all non-zero values of the constant m, the curve $y = \mathrm{e}^{mx}(x^2 + mx)$ has exactly two stationary points. [7]

6. Jan 2009 qu. 4

For each of the following curves, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and determine the exact x-coordinate of the stationary point:

(i) $y = (4x^2 + 1)^5$, [3]

(ii) $y = \dfrac{x^2}{\ln x}$. [4]

7. Jan 2009 qu. 5

The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40\mathrm{e}^{kt},$$

where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

(i) In either order,
   (a) find the values missing from the table, [3]
   (b) determine the value of k. [2]

(ii) Find the rate at which the mass is increasing when $t = 21$. [3]

8. Jan 2009 qu. 8

The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$.

The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$.
The shaded region is rotated completely about the y-axis to form a solid of volume V.

(i) Show that $V = 16\pi\left(1 - \dfrac{27}{(p+3)^3}\right)$. [6]

(ii) It is given that P is moving along the y-axis in such a way that, at time t, the variables p and t are related by

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{3}p + 1.$$

Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ at the instant when $p = 9$. [4]

9. June 2008 qu. 3

Find, in the form $y = mx + c$, the equation of the tangent to the curve $y = x^2 \ln x$ at the point with x-coordinate e. [6]

10. Jan 2008 qu. 4

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where $V = (h^6 + 16)^{\frac{1}{2}} - 4.$

(i) Find the value of $\frac{dV}{dh}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

11. Jan 2008 qu. 7

A curve has equation $y = \frac{xe^{2x}}{x+k}$, where k is a non-zero constant.

(i) Differentiate xe^{2x}, and show that $\frac{dy}{dx} = \frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k, and determine the exact coordinates of the stationary point. [5]

12. June 2007 qu. 1

Differentiate each of the following with respect to x.

(i) $x^3(x + 1)^5$ [2]

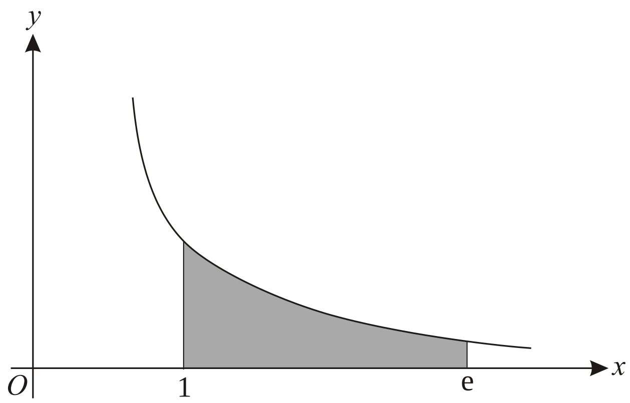
(ii) $\sqrt{3x^4 + 1}$ [3]

13. June 2007 qu. 8

(i) Given that $y = \frac{4\ln x - 3}{4\ln x + 3}$, show that $\frac{dy}{dx} = \frac{24}{x(4\ln x + 3)^2}$. [3]

(ii) Find the exact value of the gradient of the curve $y = \frac{4\ln x - 3}{4\ln x + 3}$ at the point where it crosses the x-axis. [4]

(iii)



The diagram shows part of the curve with equation

$$y = \frac{2}{x^{\frac{1}{2}}(4\ln x + 3)}.$$

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = $ e and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis.

[4]

14. Jan 2007 qu. 1

Find the equation of the tangent to the curve $y = \dfrac{2x+1}{3x-1}$ at the point $\left(1, \dfrac{3}{2}\right)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

15. Jan 2007 qu. 4

(i) Given that $x = (4t + 9)^{\frac{1}{2}}$ and $y = 6e^{\frac{1}{2}x+1}$, find expressions for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dx}$. [4]

(ii) Hence find the value of $\dfrac{dy}{dt}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]

16. Jan 2007 qu. 8

The diagram shows the curve with equation $y = x^8 e^{-x^2}$. The curve has maximum points at P and Q. The shaded region A is bounded by the curve, the line $y = 0$ and the line through Q parallel to the y-axis. The shaded region B is bounded by the curve and the line PQ.

(i) Show by differentiation that the x-coordinate of Q is 2. [5]

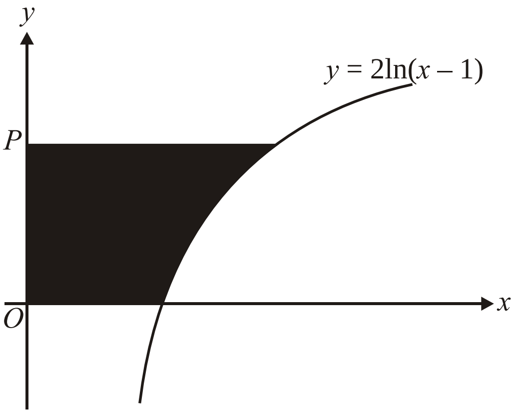
(ii) Use Simpson's rule with 4 strips to find an approximation to the area of region A. Give your answer correct to 3 decimal places. [4]

(iii) Deduce an approximation to the area of region B. [2]

17. June 2006 qu. 1

Find the equation of the tangent to the curve $y = \sqrt{4x-1}$ at the point (2, 3). [5]

18. June 2006 qu. 9



The diagram shows the curve with equation $y = 2\ln(x - 1)$. The point P has coordinates $(0, p)$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the ***y*-axis** to form a solid.

(i) Show that the volume, V cm^3, of the solid is given by $V = \pi(e^p + 4e^{\frac{1}{2}p} + p - 5)$. [8]

(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm min^{-1}. Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]

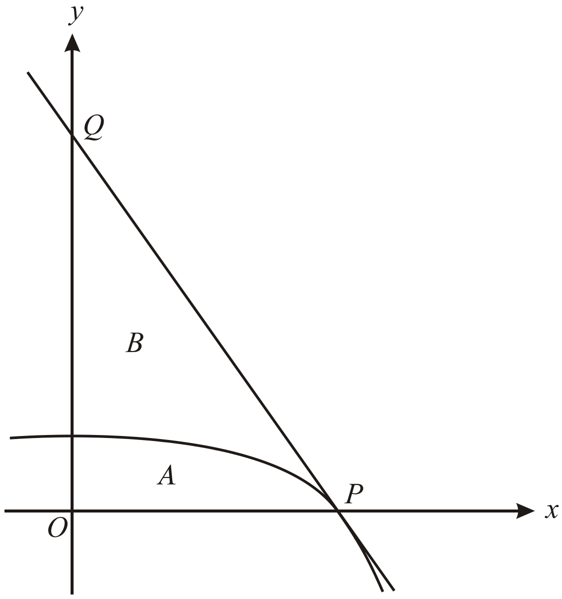
19. Jan 2006 qu. 3

(a) Differentiate $x^2 (x + 1)^6$ with respect to x. [3]

(b) Find the gradient of the curve $y = \dfrac{x^2 + 3}{x^2 - 3}$ at the point where $x = 1$.

[3]

20. Jan 2006 qu.8



The diagram shows part of the curve $y = \ln(5 - x^2)$ which meets the x-axis at the point P with coordinates (2, 0). The tangent to the curve at P meets the y-axis at the point Q. The region A is bounded by the curve and the lines $x = 0$ and $y = 0$. The region B is bounded by the curve and the lines PQ and $x = 0$.

(i) Find the equation of the tangent to the curve at P. [5]

(ii) Use Simpson's Rule with four strips to find an approximation to the area of the region A, giving your answer correct to 3 significant figures. [4]

(iii) Deduce an approximation to the area of the region B. [2]

21. June 2005 qu.6

(a) Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$. [4]

(b) The equation of a curve is $y = \dfrac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]